Marietta City Schools

2023–2024 District Unit Planner

Honors Advanced Algebra: Concepts & Connections

Unit title	Unit 2: Exponential and Logarithmic Functions	**Unit duration (hours)**	*22.5 hours*

Mastering Content and Skills through INQUIRY (Establishing the purpose of the Unit): *What will students learn?*

GA DoE Standards
Standards **AA.FGR.3** Explore and analyze structures and patterns for exponential and logarithmic functions and use exponential and logarithmic expressions, equations, and functions to model real-life phenomena. **AA.FGR.3.1** Find the inverse of exponential and logarithmic functions using equations, tables, and graphs, limiting the domain of inverses where necessary to maintain functionality, and prove by composition or verify by inspection that one function is the inverse of another. **Strategies and Methods** • To verify by inspection, students can compare graphs of two relations and show that one is the reflection of the other across the line y = x. • To verify by inspection, students can show that one table or set of coordinates is the inverse of another because the yvalues of the first are the x-values of the second and viceversa. • To verify by inspection, students can show that a series of operations upon input values of one function are opposite and reversed in order for a second function. • Students should be able to prove by composition that two functions are inverses of each other. **Terminology** • To prove by composition means to determine if $f(g(x)) = g(f(x)) = x$. **AA.FGR.3.2** Analyze, graph, and compare exponential and logarithmic functions. **Fundamentals** • Students should be able to graph and identify key features of exponential and logarithmic functions, including domain, range, and x- and y-intercepts; roots, zeros, and solutions; asymptotes; interval(s) where the function is positive, and/or negative; non-symmetry; end behavior. • Students should be able to calculate the average rate of change for a given interval, including the estimated rate of change. • Students should have opportunities to gain an intuitive sense into what happens to the graph or model as a result of changes to the various key features of the function. **AA.FGR.3.3** Use the definition of a logarithm, logarithmic properties, and the inverse relationship between exponential and logarithmic functions to solve problems in context. **Fundamentals** • Students should be given opportunities to solve real-life, culturally relevant problems involving the use of the common logarithm and the natural logarithm.

- Students should be able to apply their knowledge of the inverse relationship between exponential and logarithmic functions to solve real-life problems.

AA.FGR.3.4 Create exponential equations and use logarithms to solve mathematical, applicable problems for which only one variable is unknown.

Fundamentals

- Students should be able to solve problems involving exponential equations using the relationship with logarithmic functions to solve for the single unknown variable.
- Given pertinent information (e.g., ambient temperature and time), students should be able to use exponential equations to solve real-life problems and interpret the solutions.

Examples

- Students can solve and interpret equations that have one unknown variable, such as:
  - o Exponential growth
  - o Compound interest
  - o Newton's Law of Cooling: $T(t) = T_S + (T_0 - T_S)\, e^{-kt}$

AA.FGR.3.5 Create and interpret logarithmic equations in one variable and use them to solve problems.

Fundamentals

- Given pertinent information, students should be able to use logarithmic equations to solve real-life problems and interpret the solutions.

Example

- Students can create and interpret equations involving pH, such as pH = - log(H*), to define the acidity or alkalinity of a substance.

AA.FGR.3.6 Create, interpret, and solve exponential equations to represent relationships between quantities and analyze the relationships numerically with tables, algebraically, and graphically.

Fundamentals

- Students should be able to analyze what is happening in the relationships between quantities.
- Students should discuss the characteristics of exponential functions in context, including domain and range, zeros, intercepts, average rate of change, asymptote, and other relevant key features.
- Students should be able to solve real-life problems that can be modeled by exponential equations.
- Students should be encouraged to explore multiple solution pathways, which might include graphing with various tools, interpreting key features, and evaluating equations.

Examples

- Students can create, interpret and solve equations that have two unknown variables, such as:
  - o Half-Life
  - o Exponential growth
  - o Exponential decay
  - o Compound interest
  - o Newton's Law of Cooling: $T(t) = T_S + (T_0 - T_S)\, e^{-kt}$

AA.FGR.3.7 Create, interpret, and solve logarithmic equations in two or more variables to represent relationships between quantities.

Fundamentals

- Students should be able to analyze and interpret logarithmic equations presented in mathematical, applicable situations.
- Students should discuss the characteristics of logarithmic functions in context, including domain and range, zeros, intercepts, average rate of change, asymptote, and other relevant key features.
- Students should be able to solve problems that can be modeled by logarithmic equations.
- Students should be encouraged to explore multiple solution pathways, which might include graphing with various tools, interpreting key features, and evaluating equations.

Example

- Students are able to create and interpret equations involving logarithms such as the equation for the magnitude of an earthquakes $M = \log_{10}(I/S)$.

AA.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

AA.MM.1.1 Explain applicable, mathematical problems using a mathematical model.

Fundamentals

- Students should be provided with opportunities to learn mathematics in the context of culturally relevant problems.
- Mathematically applicable problems are problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (i.e., model with mathematics).

AA.MM.1.2 Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

Fundamentals

- Mathematically proficient students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.

AA.MM.1.3 Using abstract and quantitative reasoning, make decisions about information and data from a mathematical, applicable situation.

Fundamentals

- Students should be able to:
  - o analyze functions, graphs, tables, and equations and make decisions about the real-life situations they describe based upon their understanding of mathematical functions.
  - o analyze statistical results to decide the best course of action or approach to a problem.

Example

- Given a rectangle with length = (x – 2) and width = (2x + 3), a student could discover and articulate that the area = (x – 2)(2x + 3) = 2x2 – x – 6. From the student's understanding of parabolas, a student would know that the parabola that represents all possible areas of this rectangle opens upwards and that there is no maximum area possible for this rectangle.

AA.MM.1.4 Use various mathematical representations and structures to represent and solve real-life problems.

Fundamentals

- Students should be able to generate models, graphs, charts, and equations, to represent real-world phenomena in order to solve problems.
- Students should be provided opportunities to generate representations of real-world phenomena utilizing technology to show these phenomena and to solve problems.

Concepts/Skills to support mastery of standards

-Inverses
-Graphing Log/Exponential Functions: Characteristics and Transformations
-Create, interpret and solve exp/log (one and two variables).
-Tables of exp/log
- Properties of Logs
-Real world application

Vocabulary

Antilogarithm	Common Logarithm	Compounding	Doubling Time	Exponential Decay
Exponential Growth	One-to-One Functions	Compound Interest		

Notation

Natural log: ln   Common log: log Log_{base} (argument) = exp

Essential Questions
What is the inverse of a logarithmic function? What is the inverse of an exponential function? What are the characteristics of an exponential graph? What are the characteristics of a logarithmic graph? Which logarithmic properties are used to solve logarithmic equations? How is exponential growth/decay represented on a graph? In context?

Assessment Tasks *List of common formative and summative assessments.*
<u>Formative Assessment(s):</u> Unit Quiz, HW quizzes **<u>Summative Assessment(s):</u>** Unit Test

<u>Learning Experiences</u>		
Objective or Content	**Learning Experiences**	**Personalized Learning and Differentiation**
AA.MM.1.2 Create mathematical models to explain phenomena that exist in real world contexts. AA.FGR.3	**Investigating Exponential Functions Engage-Explore-Apply** Investigating Exponential Functions gives students a visual way to experience exponen The lesson begins with students doing a folding activity with a sheet of paper. As students continue to fold the paper, the process shows simultaneous exponential growth (the number of sections) and exponential decay (the area of one of the	Technology - Desmos, TI graphing calculators Collaborative groups

• AA.FGR.3.2 Analyze, graph, and compare exponential and logarithmic functions.	sections). In this task, students also graph a variety of exponential functions to determine the characteristics of these functions. This task gives students a visual way to experience exponential growth, as well as graphing exponential equations. The fast increase in the rate of growth is explored. Students also begin developing an understanding of compound interest - in that the original amount (principal) remains PLUS additional. The comparison of large vs. small beans as "Zombies" allows them to speculate as to how the relative size of the "zombie" might impact the total number of people who become infected with the zombie virus. **Learning Goals:** ● I can identify a relationship that is modeled by exponential decay. ● I can identify a relationship that is modeled by exponential growth. ● I can construct an exponential function that models a situation. ● I can make interpretations from an exponential model.	
AA.MM.1.2 Create mathematical models to explain phenomena that exist in real world contexts AA.FGR.3 • AA.FGR.3.1 Find the inverse of exponential and logarithmic functions using equations, tables, and graphs, limiting the domain of inverses where necessary to maintain functionality, and prove by composition or verify by inspection that one function is the inverse of another.	**Composition of Functions –Engage, Explore, Apply** In this activity, students will calculate inverse operations, identify one-to-one functions, find inverse functions symbolically, and use other representations to find inverse functions. The Engage Activity introduces students to composite functions without formally acknowledging function composition. Students are simply asked to determine whether they have enough money to purchase items from a store. There are multiple paths of arriving at the correct solution. As you allow students to share their work, you can introduce the idea of there being two functions to model this situation, namely, $f(x) = 0.75x$ which represents the cost of the items after the discount and $g(x) = 1.07x$ which represents the cost of the items on sale after including sales tax. Therefore, students can arrive at the solution by computing $g(f(x))$. In the Explore Activity, students build their own composite function by expressing regularity in repeated reasoning. First, students consider how the length of a pool determines the number of tiles that are necessary to create a border around the pool. Algebraically, there are many ways to come up with this expression; encourage students to use color to demonstrate how they "see" the tiles being added. This is a great opportunity to talk about the equivalence of expressions! After determining the number of tiles, students determine the cost of those tiles with the included delivery fee. As you monitor groups, ask students questions like "what determines the cost of the project?" or "how/why would increasing the length of the pool affect the cost?" Students should articulate that the number of tiles determines the cost, but the length of the pool determines the number of tiles. Listen for phrases like "increasing the length of the pool increases the number of tiles, which then increases the cost of the project". This kind of sequential reasoning	Technology - Desmos, TI graphing calculators Collaborative groups

	is critical for developing the students' understanding of composite functions. The Apply Activity is a numerical representation of two functions relating the depletion of the ozone layer and UV radiation and UV radiation and skin cancer. Students must apply their understanding of function composition in the context of the situation to interpret the value of their results. **Learning Goals:** ● I can calculate the composition of functions numerically. ● I can calculate the composition of functions graphically. ● I can calculate the composition of functions symbolically. ● I can verify two functions are inverses by function composition.	

Content Resources

Textbook Correlation: enVision A|G|A - Algebra 2

AA.FGR.3.1 - Lessons 6-4
AA.FGR.3.2 - Lessons 6-1, 6-4
AA.FGR.3.3 - Lessons 6-3
AA.FGR.3.4 - Lesson 6-3, 6-6
AA.FGR.3.5 - Lesson 6-5
AA.FGR.3.6 - Lessons 6-1, 6-2, Topic 6-Mathematical Modeling in 3 Acts
AA.FGR.3.7 - Lesson 6-4